Absorption and Stripping-factor Functions for Distillation Calculation by Manual- and Digital-computer Methods

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New absorption- and stripping-factor functions have been developed for use in computing multicomponent separations in fractionators, reboiled absorbers, refluxed strippers, and columns with side-stream strippers. Charts of fraction not absorbed vs. A factor and fraction not stripped vs. S factor are included with the equations for these operations.

These functions and procedures are of particular advantage in computing complex columns, i.e., two or more feeds and three or more products, where the proposed A and S factor equations provide a method for converging on a solution.

Distillation calculations for multicomponent mixtures are so tedious that process engineers have long been seeking a labor-saving method or device that will give them the required answers rapidly and with a minimum of effort. The high-speed digital computer is such a device. Its use in solving distillation problems requires careful and complete programing to obtain an automatic machine operation that duplicates rigorous manual calculations.

Experience with manual plate-to-plate and short-cut calculations is valuable in developing the machine programs. Absorption and stripping factors have been used in short-cut and in manual plate-to-plate calculations and are also useful in digital-computer distillation calculations. An advantage in using A and S factors in machine calculations is that the mathematical language is already familiar to process engineers. This makes it easier to interpret machine results and to extrapolate to different conditions as the same equations are used in hand and machine calculations.

Absorption and stripping factors have been used in distillation calculations for many years (*1* to *10*). New and more convenient relationships of these factors were required for present purposes. General functions of the A and S factors are developed and arranged for application to four distillation operations frequently used in the petroleum industry, i.e., (*a*) fractionator, (*b*) reboiled absorber, (*c*) refluxed strippers, and (*d*) fractionators with side-stream strippers. The fact that the same functions of A and S may be applied to all these columns manually or by machine is advantageous.

BASIC CONCEPTS

Component distributions in multicomponent distillation are determined by the number of theoretical stages, the interstage vapor and liquid quantities, and the vapor-liquid K values. Three of these variables are frequently grouped in absorption ($A = L/KV$) or stripping ($S = KV/L$) factors. Functions of these factors and the number of stages may be used to compute the component distributions.

Calculation methods using absorption and stripping factors may be rigorous or approximate depending upon the evaluation of the separation functions. These functions of A and S for separation calculations will be derived for two evaluation procedures: (*a*) plate-to-plate and (*b*) effective factors. Both of these may be done by hand or with a computer.

For design-calculation purposes it is convenient to consider all multistage separation processes as combinations of five separation zones: (1) condensing, (2) absorbing, (3) feed flash, (4) stripping, and (5) reboiling. Zones 1, 3, and 5 are each single-stage equilibrium flash separations, and zones 2 and 4 are multistage separations. There are two feeds to each of these multistage zones, i.e., the liquid entering the top and the vapor entering the bottoms. Likewise, there are two products: the liquid from the bottom and the vapor from the top.

It is convenient to regard both of these as separation operations where the liquid feed is subject to stripping by the rising vapors and where the vapor feed is subject to absorption by liquid flowing down the column. Thus zones 2 and 4 have both absorption and stripping taking place simultaneously, with absorption predominating in 2 and stripping in 4.

Absorption calculations are made with functions of the absorption factor, and stripping calculations are made with functions of the stripping factor for the same sections of plates. Values of A and S on individual plates may be used in some cases, but effective values of A and S for sections of plates are generally used in manual calculations. Effective values of A and S are also useful in the iterations of digital computer calculations which must be revised between trials.

COMPONENT-DISTRIBUTION EQUATIONS

Rigorous functions of absorption and stripping factors are derived for multicomponent distribution in the four basic multistage sections that constitute distillation columns: absorber, enricher, stripper, and exhauster.

Absorber (Figure 1)

A component material balance around the top of the absorber to include plates 1 through i gives

$$l_i + v_1 = v_{i+1} + l_0$$

Combining this with the equilibrium relation $l_{i+1} = A_{i+1}v_{i+1}$ and rearranging gives

$$l_{i+1} = (l_i - l_0 + v_1)A_{i+1} \qquad (1)$$

Equation (1) is the basic relationship by which the liquid leaving plate $i + 1$ is found from the liquid leaving plate i.

An equation for the multistage absorption operation is obtained by combining relationships similar to Equation (1) for each plate:

$$\begin{aligned} l_n &= v_1(A_1A_2A_3 \cdots A_n \\ &+ A_2A_3 \cdots A_n + A_3 \cdots A_n \\ &+ \cdots + A_n) - l_0(A_2A_3 \cdots A_n \\ &+ A_3 \cdots A_n + \cdots + A_n) \end{aligned} \qquad (2)$$

Equation (2) will be used in later developments.

Enricher (Figure 1)

A component material balance around the top of the enricher to include the condenser and Plate i gives

$$l_i + d = v_{i+1}$$

Combining with the equilibrium relation $l_{i+1} = A_{i+1}v_{i+1}$ and rearranging gives

$$l_{i+1} = [l_i + d]A_{i+1} \qquad (3)$$

Equation (3) is the basic relationship between the liquid leaving plate $i + 1$ and plate i. For the partial condenser, indicated in Figure 1, this relationship becomes

$$l_0 = dA_0 \qquad (3a)$$

where

$$A_0 = \frac{L_0}{DK_0}$$

For a total condenser, where reflux and distillate are of same composition, l_0/d equals the reflux ratio, and so $A_0 = R_{LD}$.

An equation for the multistage enriching operation shown in Figure 1 is obtained by combining Equation (3*a*) and relationships like Equation (3) for each stage, to obtain

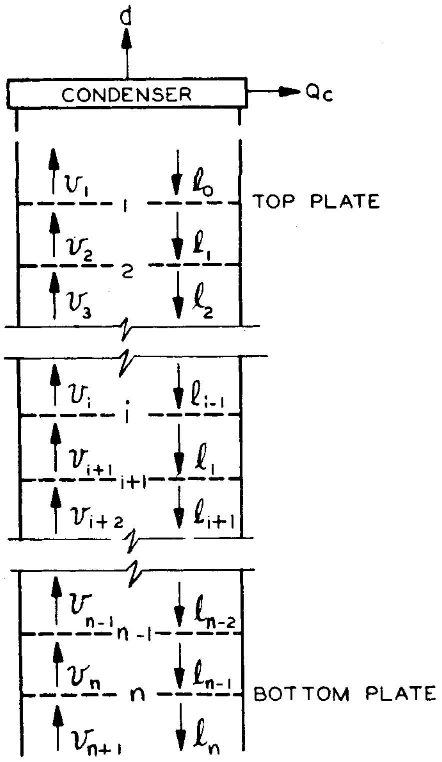

Fig. 1. Absorber (without condenser) and enricher (with condenser).

$$l_n = d(A_0A_1A_2A_3 \cdots A_n + A_1A_2A_3 \cdots A_n + A_2A_3 \cdots A_n + \cdots + A_n) \tag{4a}$$

or

$$v_{n+1} = d(A_0A_1A_2A_3 \cdots A_n + A_1A_2A_3 \cdots A_n + A_2A_3 \cdots A_n + \cdots + A_n + 1) \tag{4b}$$

Stripper (Figure 2)

A component material balance around the bottom of the stripper to include plates 1 through j gives

$$v_j + l_1 = l_{j+1} + v_0$$

Combining with the equilibrium relation $v_{j+1} = S_{j+1}l_{j+1}$ and rearranging gives

$$v_{j+1} = [v_j - v_0 + l_1]S_{j+1} \tag{5}$$

Equation (5) is the basic relationship between the vapor leaving plates $j + 1$ and j.

An equation for the multistage stripping operation is obtained by combining relationships similar to Equation (5) for each plate, to give

$$v_m = l_1(S_1S_2S_3 \cdots S_m + S_2S_3 \cdots S_m + S_3 \cdots S_m + \cdots + S_m) - v_0(S_2S_3 \cdots S_m + S_3 \cdots S_m + \cdots + S_m) \tag{6}$$

Equation (6) will be used in later developments.

Exhauster (Figure 2)

A component material balance around the bottom of the exhauster, to include the reboiler and plate j, gives

$$v_j + b = l_{j+1}$$

Combining with the equilibrium relation $v_{j+1} = S_{j+1}l_{j+1}$ and rearranging gives

$$v_{j+1} = [v_j + b]S_{j+1} \tag{7}$$

Equation (7) is the basic relation between the vapors leaving plates j and $j + 1$. For the partial reboiler, indicated in Figure 2, this relationship becomes

$$v_0 = bS_0 \tag{7a}$$

where

$$S_0 = \frac{V_0K_0}{B}$$

For a total reboiler, where the boilup vapors and the bottoms product are of the same composition, v_0/b equals the boilup ratio, and so $S_0 = R_{VB}$.

An equation for the multistage exhausting operation shown in Figure 2 is obtained by combining Equation (7a) and relationships like Equation (7) for each stage, which gives

$$v_m = b(S_0S_1S_2S_3 \cdots S_m + S_1S_2S_3 \cdots S_m + S_2S_3 \cdots S_m + \cdots + S_m) \tag{8a}$$

or

$$l_{m+1} = b(S_0S_1S_2S_3 \cdots S_m + S_1S_2S_3 \cdots S_m + S_2S_3 \cdots S_m + \cdots + S_m + 1) \tag{8b}$$

Equations (8a) and (8b) will be used in later developments.

RECOVERY FRACTIONS

Equations (2), (4), (6), and (8) are put into a more general form by rearranging, the physical significance of the absorption and of the stripping-factor functions being recognized.

When the absorption factor series $\Sigma_A = A_1A_2A_3 \cdots A_n + A_2A_3 \cdots A_n + \cdots + A_n$ and the product as $\pi_A = A_1A_2A_3 \cdots A_n$ are defined, Equation (2) becomes

$$l_n = v_1\,\Sigma_A - l_0(\Sigma_A - \pi_A) \tag{9}$$

The numerical values of Σ_A and π_A range from very small numbers (low A and high n values) to very large numbers (high A and high n values). This makes Equation (9) difficult to use for the lighter and heavier nondistributed components. Accordingly, this relationship is rearranged after it is combined with an over-all component material balance for the entire absorber and gives

$$v_1 = v_{n+1}\left(\frac{1}{\Sigma_A + 1}\right) + l_0\left[1 - \left(\frac{\pi_A}{\Sigma_A + 1}\right)\right] \tag{10}$$

By definition of two new absorption factor functions, Equation (10) becomes

$$v_1 = v_{n+1}\phi_A + l_0\psi_A \tag{10a}$$

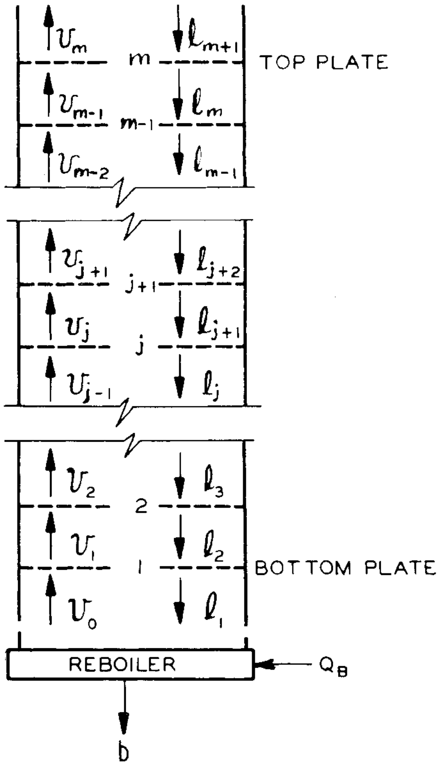

Fig. 2. Stripper (without reboiler) and exhauster (with reboiler).

An examination of these two new absorption-factor functions shows that their numerical values will always be between zero and unity, regardless of the values of A and the number of plates. These limits make Equation (10a) applicable to all components.

Equation (10a) gives the amount of any component in an absorber off gas, v, as the sum of the unabsorbed portion of the wet gas feed, $\phi_A v_{n+1}$, and the amount stripped from the lean oil, $\psi_A l_0$. Thus, ϕ_A is the fraction not recovered and $1 - \phi_A$ is the fraction recovered from the wet gas to an absorber. Likewise, ψ_A is the fraction of any component in the lean oil lost to the off gas.

By an analogous procedure a similar equation was obtained from Equation (6) for a stripper:

$$l_1 = l_{m+1}\phi_S + v_0\psi_S \tag{11}$$

where ϕ_S and ψ_S are functions of S and m.

Equation (11) gives the amount of any component in the effluent lean oil, l_1, as the sum of the unstripped portion of the rich oil feed, $\phi_S l_{m+1}$, and the amount absorbed from the stripping gas, $\psi_S v_0$. Thus ϕ_S is the fraction of any component not recovered from the rich oil to a stripper. Likewise, ψ_S is the fraction of any stripping-gas component lost to the lean oil.

An examination of Equations (10) and (11) and of the physical significance of each term reveals that

$$\psi_A = 1 - \phi_S$$

and

$$\psi_S = 1 - \phi_A$$

In other words both ψ_A and $(1 - \phi_S)$ represent the fraction of a component in the liquid feed to the top of a section of plates that is stripped off into the residue gas leaving the top. Likewise, ψ_S and $(1 - \phi_A)$ represent the fraction of a component in the gas feed to the bottom of a section of plates that is absorbed into the rich oil leaving the bottom. Thus a given section of plates is regarded as an absorber for finding ϕ_A and ψ_A and as a stripper for finding ϕ_S and ψ_S.

The foregoing can be shown to be mathematically equivalent. For simplicity take a three-plate absorber, where the plates are identified by subscripts a, b, and c from the top. From definitions of ψ_A and ϕ_S

$$\psi_A = 1 - \frac{\left(\frac{L}{KV}\right)_a\left(\frac{L}{KV}\right)_b\left(\frac{L}{KV}\right)_c}{\left(\frac{L}{KV}\right)_a\left(\frac{L}{KV}\right)_b\left(\frac{L}{KV}\right)_c + \left(\frac{L}{KV}\right)_b\left(\frac{L}{KV}\right)_c + \left(\frac{L}{KV}\right)_c + 1}$$

$$\phi_S = \frac{1}{\left(\frac{KV}{L}\right)_c\left(\frac{KV}{L}\right)_b\left(\frac{KV}{L}\right)_a + \left(\frac{KV}{L}\right)_b\left(\frac{KV}{L}\right)_a + \left(\frac{KV}{L}\right)_a + 1}$$

Writing $\psi_A = 1 - \phi_S$ in terms of the above A and S factor functions and clearing fractions proves the equivalencies.

These identities permit writing Equations (10) and (11) in terms of the ϕ fraction:

$$v_1 = \phi_A v_{n+1} + (1 - \phi_S) l_0 \qquad (12)$$

$$l_1 = \phi_S l_{n+1} + (1 - \phi_A) v_0 \qquad (13)$$

where

$$\phi_A = \frac{1}{A_1A_2A_3 \cdots A_n + A_2A_3 \cdots A_n + A_3 \cdots A_n + \cdots + A_n + 1} = 1 - f_a$$

f_a = fraction absorbed

$$\phi_S = \frac{1}{S_1S_2S_3 \cdots S_m + S_2S_3 \cdots S_m + S_3 \cdots S_m + \cdots + SS_m + 1} = 1 - f_s$$

f_s = fraction stripped

Plates are numbered from the top down in evaluating the ϕ_A functions for both equations and from the bottom up in evaluating the ϕ_S functions.

Equations (12) and (13) are generalized separation functions that express the physical operation in terms of fractions absorbed and stripped. These fractions are functions of the absorption and stripping factors.

These functions are applicable to sections of any type of column, i.e., absorber, stripper, fractionator, etc. The values of ϕ_A and ϕ_S are always between zero and unity regardless of the magnitude of the A and S values. This is an advantage in evaluating and using the functions.

ENRICHING AND EXHAUSTING EQUATIONS

Component distribution functions of ϕ_A and ϕ_S for the enriching and exhausting sections (Figures 1 and 2) of a fractionator may be derived from Equations (4) and (8) and the definitions of ϕ_A and ϕ_S. Alternatively they may be derived from Equations (12) and (13) plus component material balances around the condenser and the reboiler.

An enriching equation may be derived from the absorber equation [Equation (12)] by making the lean oil equal zero, i.e., $l_0 = 0$, and letting the top plate be a partial condenser. Equation (12) then reduces to Equation (4*b*). In like manner Equation (8*b*) may be obtained from Equation (13). Development of enriching and exhausting equations by combination follows.

An equation for the entire enriching section is readily obtained from Equation (12) by replacing v_1, v_{n+1}, and l_0 by their equivalents:

$v_1 = l_0 + d$ (component material balance around condenser)

$l_0 = A_0 d$ (vapor-liquid equilibria for partial condenser)

$v_{n+1} = l_n + d$ (over-all component material balance)

Combining and rearranging gives

$$\frac{l_n}{d} = \frac{A_0\phi_{SE} + 1}{\phi_{AE}} - 1 \qquad (14a)$$

or

$$\frac{v_{n+1}}{d} = \frac{A_0\phi_{SE} + 1}{\phi_{AE}} \equiv \frac{A_0\phi_{SE}}{\phi_{AE}} + \frac{1}{\phi_{AE}} \qquad (14b)$$

where the second subscript E designates the enriching section. For a total condenser $A_0 = L_0/D$, the reflux to distillate ratio.

Equation (14) gives the ratio of the amount of any component in the liquid leaving the bottom of the enriching section to the amount in the distillate. This will be called the "enriching ratio." The functions ϕ_{SE} and ϕ_{AE} will be between zero and unity for all components.

A similar equation may be written for the exhausting section by combining Equation (13) with the equivalents:

$l_1 = v_0 + b$ (component material balance around reboiler)

$v_0 = S_0 b$ (vapor-liquid equilibria for partial reboiler operation)

$l_{m+1} = v_m + b$ (over-all component material balance)

This combination, followed by rearranging, gives

$$\frac{V_m}{b} = \frac{S_0\phi_{AX} + 1}{\phi_{SX}} - 1 \qquad (15a)$$

or

$$\frac{l_{m+1}}{b} = \frac{S_0\phi_{AX} + 1}{\phi_{SX}} \equiv \frac{S_0\phi_{AX}}{\phi_{SX}} + \frac{1}{\phi_{SX}} \qquad (15b)$$

where the second subscript X designates the exhausting section.

When the reboiler vapor is of the same composition as bottoms product, $S_0 = V_0/B$ = boilup ratio.

Equation (15) gives the "exhausting ratio" as a function of this boilup ratio and the functions ϕ_{AX} and ϕ_{SX}, which are also between 0 and 1.

Equations (14*a*) and (4*a*) both give l_n/d as a function of the L, V, and K values for each plate. The different arrangements of the variables make these two equations suitable for different calculation purposes, but both are useful. Likewise, Equations (15*a*) and (8*a*) both give n_m/b as a function of the L, V, and K values for each plate. These equations are also used for different kinds of calculations. The applications of these four equations will be taken up further after "effective" A and S factors are defined and the ϕ_A and ϕ_S functions evaluated.

EFFECTIVE FACTORS

The recovery fractions, ϕ_A and ϕ_S in Equations (12), (13), (14), and (15), may be evaluated from assumed or previous trial values of K, V, and L on each plate. When the correct values of A and S are used, this method is rigorous. In some calculations (by hand and machine) it is convenient to use effective absorption and stripping factors, A_e and S_e. A_e is a mean value of the absorption factor that will give the same value of ϕ_A that is obtained from using the values of A on each plate. A_e is defined as follows:

$$\phi_S = \frac{1}{A_e^{\,n} + A_e^{\,n-1} + \cdots + A_e^{\,2} + A_e + 1} = \frac{A_e - 1}{A_e^{\,n+1} - 1} \qquad (16a)$$

For $A_e = 1.0$ the value of ϕ_A is

$$\phi_A = \left[\frac{1}{1+n}\right]_{A=1.0}$$

For an infinite number of plates and $A \geq 1.0$, the value of ϕ_A is zero.

$$\phi_A = [0]_{A\geq 1.0,\ n=\infty}$$

For infinite plates and less-than-unity A factors

$$\phi_A = [1 - A]_{A<1.0,\ n=\infty}$$

Analogous (effective) stripping-factor relationships can be written by inspection; i.e.,

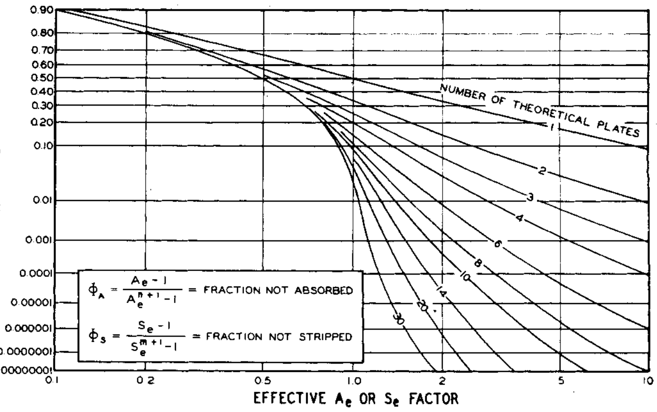

Fig. 3. Functions of absorption and stripping factors.

$$\phi_S = \frac{1}{S_e^{\ m} + S_e^{\ m-1} + \cdots + S_e^{\ 2} + S_e + 1} = \frac{S_e - 1}{S_e^{\ m+1} - 1} \qquad (16b)$$

$$\phi_S = \left[\frac{1}{1+m}\right]_{S=1.0}$$

$$\phi_S = [0]_{S\geq 1.0,\ m=\infty}$$

$$\phi_S = [1 - S]_{S<1.0,\ m=\infty}$$

Figure 3 is a plot of ϕ_A vs. A_e and n and of ϕ_S vs. S_e and m. A probability scale is used for ϕ_A and ϕ_S and a logarithmic scale for A_e and S_e. This chart and Figures 3A and 3B were prepared from Equation (16). Rectangular coordinates for ϕ_A and A_e would give a plot similar in appearance to the Kremser-Brown absorption-factor–vs.–fraction (or per cent) extraction plot. (*See* Figure 3A.) Figure 3B is an expanded plot on logarithmic scales of the lower right-hand corner of Figure 3A. Hull and Raymond (*6*) used such a Kremser-Brown chart, plotting $(A^{n+1} - A)/(A^{n+1} - 1) = 1 - (A - 1/A^{n+1} - 1)$.

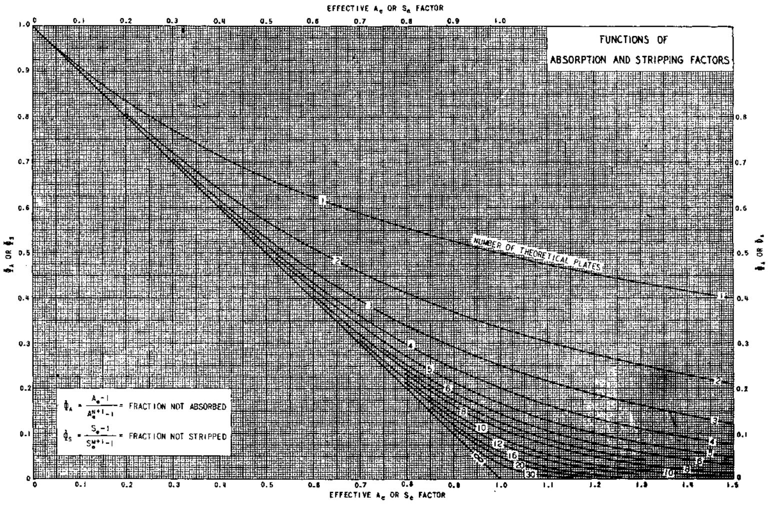

Fig. 3*a*. Functions of absorption and stripping factors.

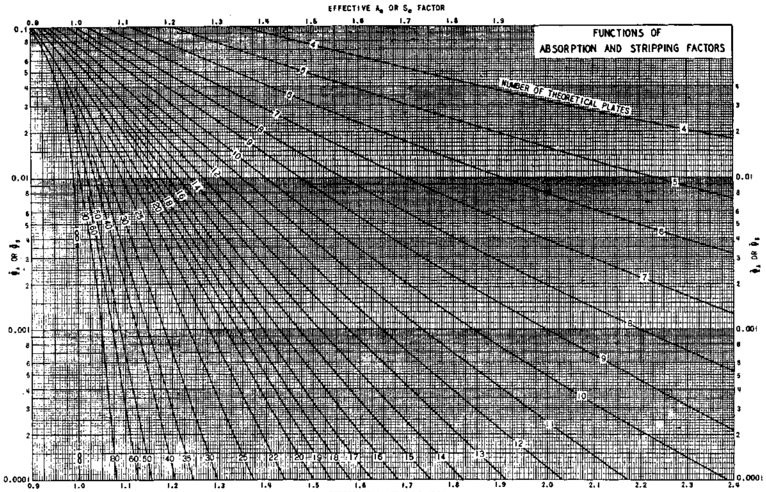

Fig. 3*b*. Functions of absorption and stripping factors.

For two plates the effective factors are given by the relations previously proposed (*2*).

$$A_e = \sqrt{A_B(A_T + 1) + 0.25} - 0.5 \qquad (17a)$$

$$S_e = \sqrt{S_T(S_B + 1) + 0.25} - 0.5 \qquad (17b)$$

Subscripts B and T designate the bottom and top plates, respectively. In many cases the effective factors are functions of terminal (top and bottom plates) conditions only and independent of the number of plates. In these cases correct values of A_e and S_e are obtained by these simple relations. Even where this simplification is not justified for final calculation, it may be used as a first approximation.

In some calculations both A_e and S_e are required for the same group of plates. For absorption factors the plates are numbered from the top down, and for stripping factors from the bottom up. For this reason $A_1 = A_T$ and $S_1 = S_B$ (subscripts T and B refer to top and bottom). Also $S_T = 1/A_T$ and $S_B = 1/A_B$.

Another method of finding the values of A_e and S_e is by a proportionality relationship based upon the method proposed by Horton and Franklin (*5*) for locating the effective factor. The location of the effective factor depends upon the value of A (or S). Assuming a linear relationship between A (or S) and the number of theoretical plates and using the locations given by Horton and Franklin, one can write the following equations for A_e and S_e.

Value of A (or S)	$A_e =$	$S_e =$
0 to 0.1	A_B	S_T
0.1 to 0.4	$A_B + 0.1\,(A_T - A_B)$	$S_T + 0.1\,(S_B - S_T)$
0.4 to 1.0	$A_B + 0.2\,(A_T - A_B)$	$S_T + 0.2\,(S_B - S_T)$
1.0 to 4.0	$A_B + 0.3\,(A_T - A_B)$	$S_T + 0.3\,(S_B - S_T)$
4.0 to ∞	$A_B + 0.4\,(A_B - A_B)$	$S_T + 0.4\,(S_B - S_T)$

These proportionality relationships give essentially the same values of A_e and S_e that are found by Equation (17). It will be noted that A_e is always nearer to A_B and that S_e is always nearer to S_T. These relationships for effective factors must be modified for absorbers with intercoolers and distillation columns with intermediate reflux. For A_e and S_e they are accurate for two theoretical plates only but are convenient approximations for more plates. A rigorous solution for A_e or S_e is an iterative calculation requiring starting values of A and S and revision methods to converge on the satisfactory values.

In hand calculations the starting values may be approximated from the column terminals. For computer work plate-to-plate calculations may be made from the column terminals, starting with the desired products and the given or assumed reflux ratio. From the results of these calculations, values of A_e and S_e may be found by inverting the functions developed above.

Equation (16) may be inverted to find A_e for known values of ϕ_A and n by an iterative procedure which is readily handled on a computer. In the following equation the single primes designate any trial value of A_e and the double primes designate the A_e for the next trial.

$$A_e'' = A_e' - \left[\frac{1+(A_e')+(A_e')^2+(A_e')^3+\cdots+(A_e')^{n-1}+(A_e')^n-(1/\phi_A)}{1+2(A_e')+3(A_e')^2+\cdots+(n-1)(A_e')^{n-2}+(A_e')^{n-1}}\right] \qquad (18a)$$

This equation is from the series equation for $1/\phi_A$. The numerator of the fraction gives the difference between starting and calculated values of $1/\phi_A$. The denominator is the first derivative of the series solution for $1/\phi_A$. Solution of this equation is repeated until the value of A_e is constant for successive trials; i.e., $A_e'' = A_e'$. Cyclic solutions such as this are readily handled on digital computers.

Equation (16) may be solved for n for known values of A_e and ϕ_A by the following equation:

$$n = \frac{\log\left[\dfrac{1+\dfrac{1}{\phi_A}(A_e-1)}{A_e}\right]}{\log A_e} \qquad (19a)$$

The solution of Equation (19a) is more straightforward than the solution of Equation (18a), no cycling with successive approximations being required.

Analogous equations for S_e and m may be written by inspection. Graphical solution of these relationships are convenient for hand calculations and illustrate the functions. (See Figures 3, 3*A*, and 3*B*.)

The enriching and exhausting equations, i.e. Equations (4) and (8) and Equations (14) and (15), may also be written in terms of effective factors, giving an enriching equation in terms of A_e and an exhausting equation in terms of S_e. For the enricher

$$\frac{v_{n+1}}{d} = A_0A_e^n + A_e^n + A_e^{n-1} + \cdots + A_e^2 + A_e + 1 \qquad (20a)$$

$$\frac{v_{n+1}}{d} = A_0A_e^n + \frac{A_e^{n+1}-1}{A_e-1}$$

For the exhauster

$$\frac{l_{m+1}}{b} = S_0S_e^m + S_e^m + S_e^{m-1} + \cdots + S_e^2 + S_e + 1 \qquad (20b)$$

$$\frac{l_{m+1}}{b} = S_0S_e^m + \frac{S_e^{m+1}-1}{S_e-1}$$

Equation (20a) may be inverted to

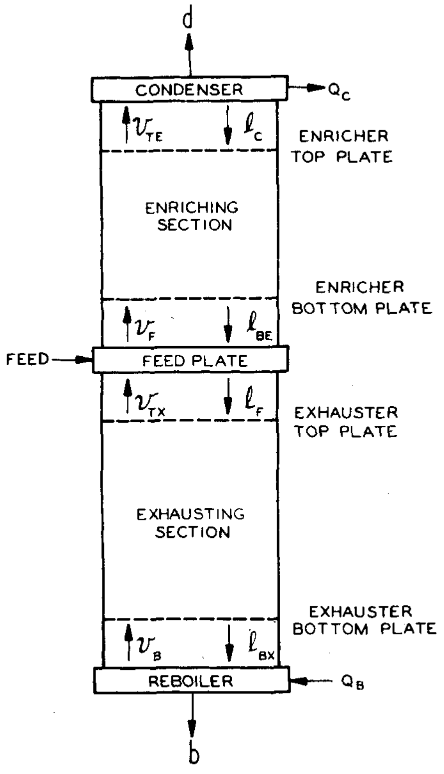

Fig. 4. Schematic diagram of fractionator.

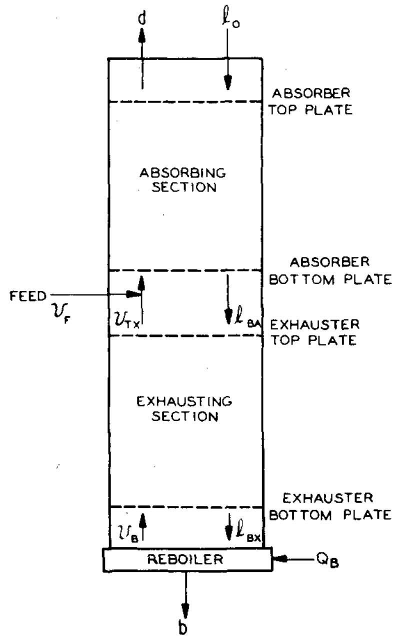

Fig. 5. Reboiled absorber.

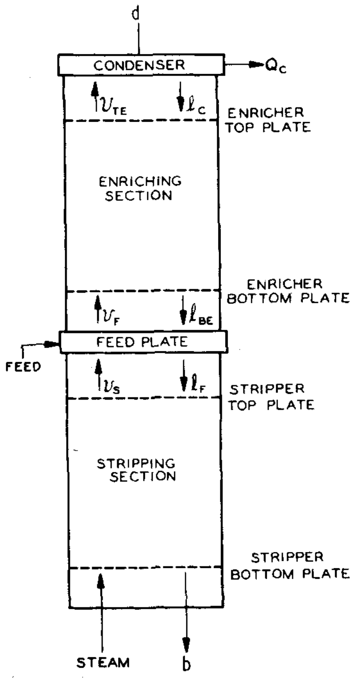

Fig. 6. Refluxed steam stripper.

give A_e in terms of v_{n+1}/d, A_0, and n by the following:

$$A_e'' = A_e' - \left[\frac{1+(A_e')+(A_e')^2+(A_e')^3+\cdots+(A_e')^{n-1}+(1+A_0)(A_e')^n-\dfrac{v_{n+1}}{d}}{1+2(A_e')+3(A_e')^2+\cdots+(n-1)(A_e')^{n-2}+n(1+A_0)(A_e')^{n-1}}\right] \tag{18b}$$

The value of n for known values of A_e, A_0, and v_{n+1}/d is found by the following equation:

$$n = \frac{\log\left[\dfrac{1+\dfrac{v_{n+1}}{d}(A_e-1)}{(1+A_0)A_e-A_0}\right]}{\log A_e} \tag{19b}$$

Analogous equations may be written for S_e and m.

The foregoing equations in effective factors are useful in making revisions in starting conditions for plate-to-plate calculations. These revisions are handled by a computer. Equations for fractionators, absorbers, and other columns are obtained by combining equations for the various sections.

FRACTIONATOR

Figure 4 is a schematic diagram of a fractional distillation column. This apparatus is obtained by combining the enricher of Figure 1 and the exhauster of Figure 2 with a feed plate, which is an equilibrium stage where vapor from the exhausting section and liquid from the enriching section join and mix with the fresh feed and then flash to give equilibrium vapor and liquid which go to enriching and exhausting sections, respectively (Figure 4).

By definition $A_F = l_F/v_F$. From this definition it follows that

$$\frac{b}{d} = A_F\left(\frac{v_F/d}{l_F/b}\right) \tag{22}$$

Separation functions for the enricher and exhauster are written in terms of l_F and v_F.

The vapor leaving the feed plate is the vapor going to plate n, and so Equation (14*b*) may be written

$$\frac{v_F}{d} = \frac{A_0\phi_{SE}+1}{\phi_{AE}} \tag{14c}$$

Likewise, the liquid leaving the feed plate is the liquid going to plate m, and so Equation (15*b*) may be written

$$\frac{l_F}{b} = \frac{S_0\phi_{AX}+1}{\phi_{SX}} \tag{15c}$$

Combining Equations (22), (14*c*), and (15*c*) gives

$$\frac{b}{d} = A_F\frac{\left(\dfrac{A_0\phi_{SE}+1}{\phi_{AE}}\right)}{\left(\dfrac{S_0\phi_{AX}+1}{\phi_{SX}}\right)} \tag{23}$$

A similar equation is obtained by combining Equations (20*a*), (20*b*), and (22) in like manner.

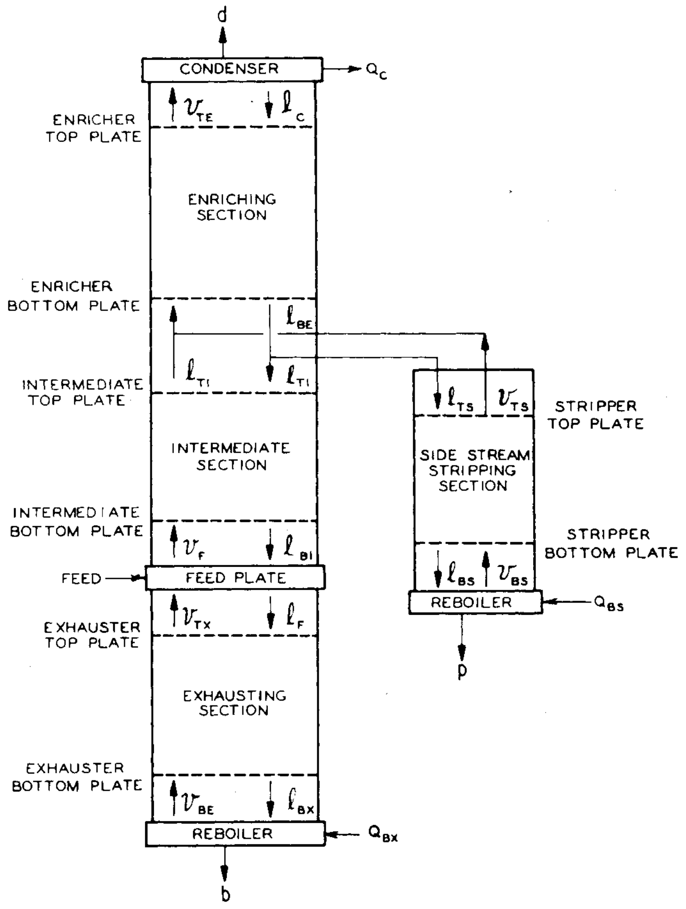

Fig. 7. Schematic diagram of fractionator with side stream.

$$\frac{b}{d} = A_F\frac{\left[A_0A_e^n+\left(\dfrac{A_e^{n+1}-1}{A_e-1}\right)\right]}{\left[S_0S_e^m+\left(\dfrac{S_e^{m+1}-1}{S_e-1}\right)\right]} \tag{24}$$

Equations (23) and (24) and the over-all component material balance, $b + d = f$, are solved with assumed temperatures and liquid-vapor traffics for a given column. The results are used to check the assumed conditions. Revised assumptions and repeated calculations may be necessary.

REBOILED ABSORBER

Figure 5 is a schematic diagram of a reboiled absorber, which is a combination of the absorber of Figure 1 and the exhauster of Figure 2. An all-vapor feed with an equilibrium flash of feed is shown. An equation for this operation is obtained by combining Equation (12) for the absorber and Equation (15*a*) for the exhauster. For this case Equation (12) may be written

$$d = \phi_{AA}(v_{TX}+v_F) + (1-\phi_{SA})l_0 \tag{25}$$

Equation (15*a*) for this case becomes

$$\frac{v_{TX}}{b} = \frac{S_0\phi_{AX}+1}{\phi_{SX}} - 1 \tag{26}$$

Combining Equations (25) and (26) and rearranging gives

$$\frac{d}{b} = \frac{\phi_{AA}\left(\dfrac{S_0\phi_{AX}+1}{\phi_{SX}}\right)+(1-\phi_{SA}-\phi_{AA})\dfrac{l_0}{b}}{(1-\phi_{AA})} \tag{27}$$

Equation (27) is solved for a given theoretical column with assumed temperature and L and V values at the key points. An over-all material balance, $d + b = l_0 + v_F$, is used with these equations to find the values of d and b that satisfy the given and the assumed conditions. Machine calculations for reboiled absorbers will utilize Equation (27) for making revisions in terminal-product assumptions.

REFLUXED STEAM STRIPPER

Figure 6 is a schematic diagram of a refluxed steam stripper. This is a combination of the enricher of Figure 1, the stripper of Figure 2, and an equilibrium feed plate.

With steam as the stripping medium $v_0 = 0$, $l_1 = b$ in Equation (13), so that $l_F = b/\phi_{SX}$ for the stripper. This relationship and the feed-plate relationship $l_F = A_F v_F$ are combined with Equation (14c) to give the following equation for this operation:

$$\frac{b}{d} = A_F\left(\frac{\phi_{SS}}{\phi_{AE}}\right)(A_0\phi_{SE} + 1) \qquad (29)$$

This equation and the over-all material balance, $b + d = f$, are solved with assumed temperatures and liquid-vapor traffics for a given column. The results are used to check starting conditions and make revisions.

FRACTIONATOR WITH SIDE STREAM STRIPPER

Figure 7 is a schematic diagram of a fractionator with a side stream stripper. This apparatus splits the feed into three products. There are four sections of plates, two reboilers, one condenser, and a feed plate.

Component distributions for this operation may be found by solving simultaneously

over-all component balance:

$$d + b + p = f \qquad (29)$$

for the feed plate:

$$l_F/v_F = A_F \qquad (30)$$

for the exhausting section:

$$\frac{l_F}{b} = \left[\frac{S_0\phi_{AX} + 1}{\phi_{SX}}\right] = \left[S_0S_e^m + \left(\frac{S_e^{m+1} - 1}{S_e - 1}\right)\right] \qquad (31)$$

$$v_{TX} = l_F - b \qquad (32)$$

for the intermediate section:

$$v_{TI} = \phi_{AI}v_F + (1 - \phi_{SI})l_{TI} \qquad (33)$$

for the side stream stripper:

$$\frac{l_{TS}}{p} = \left[\frac{S_0\phi_{AS} + 1}{\phi_{SS}}\right] = \left[S_0S_e^m + \left(\frac{S_e^{m+1} - 1}{S_e - 1}\right)\right] \qquad (34)$$

$$v_{TS} = l_{TS} - p = Xl_{BE} - p \qquad (35)$$

where X = fraction of liquid from enriching-section bottom plate going to side stream stripper.

for the enriching section:

$$\frac{l_{BE}}{d} = \left[\frac{A_0\phi_{SE} + 1}{\phi_{AE}} - 1\right] = \left[A_0A_e^n + \left(\frac{A_e^{n+1} - 1}{A_e - 1}\right) - 1\right] \qquad (36)$$

$$l_{BE} + d = v_{TS} + v_{TI} \qquad (37)$$

With the nine equations above it is possible to solve for the amounts of all components in the three products that would come from an assumed column with given temperatures and liquid and vapor traffics.

Combining these equations and rearranging gives the following relationship for the amount of any component in the distillate from its amount in the feed and the values of the different separation functions:

$$d = \frac{f}{1 + \dfrac{(l_{BE}/d)}{(l_{TS}/p)}X + \dfrac{A_F}{(l_F/b)\phi_{AI}}\left[1 + \dfrac{(l_{BE}/d)}{(l_{TS}/p)}X + (1 - X)(l_{BE}/d)\phi_{SI}\right]} \qquad (38)$$

With values of d from Equation (38) it is possible to find values of p and b and also the compositions of internal liquid and vapor streams. From the latter the initially assumed temperatures may be checked. Heat balance may then be made to check the liquid and vapor quantities.

Equations (31), (34), and (36) are double equations giving alternate ways of evaluating these terms, depending upon the problem requirements. Equations (29) through (38) have been solved by hand for an assumed column, temperature gradient, and liquid and vapor traffics. The revision of the assumptions and further trials by hand are most tedious, but iterative calculation can be readily made on a computer. These equations furnish the basis for developing such a program. Conventional plate-to-plate calculations can be made by starting with the results from these equations.

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The assistance of G. G. Bejarano in checking the mathematics is asknowledged.

NOTATION

f = moles/hr. of any component in feed
Σf = F = moles/hr. of total feed
b = moles/hr. of any component in bottoms product
Σb = B = moles/hr. of total bottoms product
d = moles/hr. of any component in distillate
Σd = D = moles/hr. of total distillate
v = moles/hr. of any component in vapor at designated point
Σv = V = moles/hr. of total vapor at designated point
l = moles/hr. of any component in liquid at designated point
Σl = L = moles/hr. of total liquid at designated point
K = y/x, by definition
y and x = mole fractions in vapor and liquid phases in equilibrium
y = v/V and $x = l/L$
A = L/KV and $S = KV/L$, by definition.
l/v = L/KV for any component at any equilibrium stage
R_{LD} = L_0D = reflux ratio
R_{VB} = V_0/B = boilup ratio
ΣA = $A_1A_2A_3 \cdots A_n + A_2A_3 \cdots A_n + \cdots + A_n$
ΣS = $S_1S_2S_3 \cdots S_m + S_2S_3 \cdots S_m + \cdots + S_m$
π_A = $A_1A_2A_3 \cdots A_n$
π_S = $S_1S_2S_3 \cdots S_n$
ϕ_A = $1/\Sigma_A + 1 = (A_e - 1)/(A_e{}^{n+1} - 1) = 1 - f_a$ = fraction of any wet-gas component not recovered in an absorber
ϕ_S = $1/\Sigma_S + 1 = (S_e - 1)/(S_e{}^{m+1} - 1) = 1 - f_s$ = fraction of any rich-oil component not recovered in a stripper
ϕ_{AE} = value of ϕ_A for enriching section
ϕ_{SE} = value of ϕ_S for enriching section
ϕ_{AX} = value of ϕ_A for exhausting section
ϕ_{SX} = value of ϕ_S for exhausting section
ψ_A = $[1 - \pi_A/(\Sigma_A + 1)]$ = fraction of any lean-oil component leaving an absorber in the off gas
ψ_S = $[1 - \pi_S/(\Sigma_S + 1)]$ = fraction of any stripping-gas component leaving a stripper in the lean oil
f_a = fraction absorbed
f_s = fraction stripped
Q_c = heat removed by condenser
Q_B = heat added in reboiler

Subscripts

i = any plate in enricher or absorber
n = bottom plate in enricher or absorber
j = any plate in exhauster or stripper
m = top plate in exhauster or stripper
∞ = pinch point

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